Fermi National Accelerator Laboratory

Spin Dynamics in Accelerators and Storage Rings[*]

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INTRODUCTION

The understanding of charged particle orbital dynamics in an electromagnetic field is necessary for the acceleration and the storage of particles. The understanding of spin (or magnetic moment) dynamics is essential for the acceleration and the storage of polarized particle beams. In this paper we review the analysis of the dynamics of the spin and the various schemes and devices proposed up-to-date to manipulate the spin to conserve the polarization while accelerating and storing a polarized beam. We shall concentrate on hadrons such as protons, antiprotons and heavier ions although most of the discussions apply equally well to leptons. Electron and positron beams are self-polarizing through spin-flip synchrotron radiation[1] at high energies and hence the acceleration of polarized $e^{\pm}$ beams is less relevant.

RELATIVISTIC SPIN EQUATION AND
DEPOLARIZING RESONANCES

In covariant form the relativistic spin equation is[2,3]

$$\frac{ds^{\alpha}}{d\tau} = \frac{e}{mc}\left[(1+G)F^{\alpha\beta}s_{\beta} + \frac{G}{c^2}u^{\alpha}s_{\lambda}F^{\lambda\mu}u_{\mu}\right] \tag{1}$$

where

τ = proper time = time in rest-frame,
s^{α} = spin 4-vector,
u^{α} = velocity 4-vector,
$F^{\alpha\beta}$ = electromagnetic field 4-tensor,
e and m = charge and mass of particle,
c = speed of light,
$G = \frac{g-2}{2}$ = anomalous gyromagnetic ratio
$= \begin{cases} 1.793 & \text{for proton} \\ 0.00116 & \text{for electron.} \end{cases}$

The 3 spatial components can be written as

$$\frac{d\vec{s}}{dt} = \frac{e}{\gamma mc} \vec{s} \times \left[(1+\gamma G)\vec{B}_\perp + (1+G)\vec{B}_\parallel + (\frac{\gamma}{\gamma+1}+\gamma G)\frac{\vec{E} \times \vec{v}}{c} \right] \qquad (2)$$

where

t = time,

$\vec{s}$ = proper spin = spin in rest-frame,

γ = energy in units of mc^2,

v = velocity,

$\vec{B}_\perp$ and $\vec{B}_\parallel$ = magnetic field perpendicular and parallel to $\vec{v}$,

$\vec{E}$ = electric field,

all quantities except $\vec{s}$ are expressed in the lab frame. In accelerators and storage rings the electric term is generally negligible. We will also transform the independent variable t to the turning angle θ around the ring by the relations

$$v dt = (1+\frac{x}{\rho}) ds \qquad \text{and} \qquad \frac{ds}{\rho} = d\theta$$

where

ρ = radius of curvature of the planar closed orbit,

s = distance along the closed orbit,

and we used the right-handed coordinates

x = normal in the horizontal plane of the closed orbit,

y = tangent to the closed orbit,

z = vertical.

The transformed equation is

$$\frac{d\vec{s}}{d\theta} = \vec{s} \times \vec{\Omega} \qquad (3)$$

where the precession vector is

$$\vec{\Omega} = \frac{\rho}{(B\rho)}(1+\frac{x}{\rho}) \left[(1+\gamma G)\vec{B}_\perp + (1+G)\vec{B}_\parallel \right]_{\text{particle}}$$

$$\equiv \varsigma_R \hat{x} - \varsigma_I \hat{y} - \kappa \hat{z} = \frac{1}{2}(\varsigma \hat{X} + \varsigma^* \hat{X}^*) - \kappa \hat{z}$$

and, as defined

$$\varsigma = \varsigma_R + i\varsigma_I \qquad \text{and} \qquad \hat{X} = \hat{x} + i\hat{y}.$$

To calculate the components ς and κ of $\vec{\Omega}$ one must first solve the orbit equation

$$\frac{d\vec{v}}{dt} = \frac{e}{\gamma mc}\, \vec{v} \times \vec{B} \tag{4}$$

with given $\vec{B}(x,z,s)$ for the orbital motion $x = x(s)$, $z = z(s)$, then obtain $(\vec{B}_\perp)_{particle}$ and $(\vec{B}_{||})_{particle}$ for use in the spin equation (3). Or one could express $\vec{B}_\perp$ and $\vec{B}_{||}$ directly in terms of the particle coordinates. The result[4] is, to first order in x and z.

$$\begin{cases} \kappa = \gamma G = \text{vertical precession angular velocity} \\ \varsigma = -\ (1+\gamma G)\rho z'' - i[(1+\gamma G)z' - \rho(1+G)(\tfrac{z}{\rho})'] \equiv \Sigma\ \epsilon e^{-i\kappa_0 \theta} \\ \qquad (\epsilon \equiv |\epsilon|e^{i\lambda}, \quad \text{and} \quad \text{prime} \equiv \tfrac{d}{ds}) \end{cases} \tag{5}$$

The (complex) horizontal precession angular velocity ς is zero for particles traveling on the ideal planar closed orbit on which the magnetic field is everywhere vertical. Thus, ς is the coupling between up and down spins responsible for depolarizing the beam. As indicated, it is multiperiodic with frequencies

$$\kappa_0 = \text{integer} = k$$

arising from vertical closed orbit distortions caused by magnet imperfections and

$$\kappa_0 = \text{superperiodicity} \pm \text{vertical oscillation tune}$$
$$= kS \pm \nu_z$$

arising from the vertical oscillation of the particle in a magnet lattice with S superperiods. There may also be frequencies involving ν_x and ν_y entering through higher order terms but they are generally small. The amplitude $|\epsilon|$ of ς has the energy dependence.

$$|\epsilon| \propto \gamma \quad \text{for ``imperfection'' terms } \kappa_0 = k$$

and

$$|\epsilon| \propto \sqrt{\gamma/\beta} \quad \text{for ``intrinsic'' terms } \kappa_0 = kS\pm\nu_z.$$

Resonance occurs when the vertical precession frequency equals one of the frequencies contained in ς, or

$$\kappa \equiv \gamma G = \begin{cases} k & \text{imperfection resonance} \\ kS \pm \nu_z & \text{intrinsic resonance} \end{cases} \tag{6}$$

On a resonance the horizontal precession may accumulate to large values and depolarization may result.

Computer programs are available to evaluate ϵ for specific ring lattices using Eq. (5). The computed resonance strengths for several synchrotrons and storage rings are plotted in Fig. 1. The γ dependencies of the imperfection and the intrinsic resonances are evident. The values of $|\epsilon|$ range from 10^{-2} at the low energy end of the AGS booster to over 10 at the high energy end of the SSC.

SPINOR FORMULATION AND RESONANCE CROSSING

To proceed further it is more convenient to use the spinor formulation. For this we write

$$\vec{s} = \psi^+ \vec{\sigma} \psi$$

where

$$\begin{cases} \psi = \psi(\theta) = \text{2-component spinor,} \\ \vec{\sigma} = \text{Pauli spin matrices,} \\ \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \end{cases}$$

and where + denotes hermitian conjugation. The spin equation (3) then becomes the spinor equation

$$\frac{d\psi}{d\theta} = \frac{i}{2} H\psi \tag{7}$$

where the "hamiltonian" is

$$H = \vec{\Omega} \cdot \vec{\sigma} = \varsigma_R \sigma_x - \varsigma_I \sigma_y - \kappa \sigma_z = \begin{pmatrix} -\kappa & \varsigma \\ \varsigma^* & \kappa \end{pmatrix}$$

This is equivalent to going from the Heisenberg picture to the Schrödinger picture in quantum mechanics. ψ is the state vector. In this formulation vertical polarization is defined as

$$P_z \equiv \langle \psi^+ \sigma_z \psi \rangle$$

For a single resonance $\varsigma = \epsilon e^{-i\kappa_o\theta}$, we transform to the resonance frame by

$$\psi = e^{-\frac{i}{2}\kappa_o\theta\sigma_z}\phi$$

and get

$$\frac{d\phi}{d\theta} = \frac{i}{2}K\phi \tag{8}$$

where

$$K = \vec{\omega}\cdot\vec{\sigma} = \epsilon_R\sigma_x - \epsilon_I\sigma_y - \delta\sigma_z = \begin{pmatrix} -\delta & \epsilon \\ \epsilon^* & \delta \end{pmatrix},$$

$$(\delta \equiv \kappa - \kappa_o).$$

If δ and ϵ are constants (or adiabatic invariants) the formal solution is

$$\phi(\theta) = e^{\frac{i}{2}\theta\vec{\omega}\cdot\vec{\sigma}}\phi(0) \equiv M\phi(0).$$

The exponential operator defined as

$$M \equiv e^{\frac{i}{2}\theta\vec{\omega}\cdot\vec{\sigma}} = e^{\frac{i}{2}(\omega\theta)(\hat{\omega}\cdot\vec{\sigma})}$$

$$\equiv \cos\frac{\omega\theta}{2} + i(\hat{\omega}\cdot\vec{\sigma})\sin\frac{\omega\theta}{2}$$

is a rigid rotation of angle $\omega\theta$ about the axis $\hat{\omega}$. In general for a succession of rotations

$$M = M_kM_{k-1}\cdots\cdots M_2M_1 \equiv e^{\frac{i}{2}\theta(\hat{n}\cdot\vec{\sigma})}$$

The equivalent single rotation angle θ and axis $\hat{n}$ are given by

$$\begin{cases} \cos\frac{\theta}{2} = \frac{1}{2}\text{Tr}(M) \\ \\ \hat{n} = \dfrac{1}{2i\sin\frac{\theta}{2}}\text{Tr}(\vec{\sigma}M) \end{cases} \tag{9}$$

The steady state solutions of Eq. (8) (eigenstates of K) give at $\delta = 0$ the polarizations

$$P_z(\delta) = \pm\frac{\delta}{\sqrt{\delta^2 + |\epsilon|^2}}$$

Instantaneous transitions of these states to $-\delta$ give

$$\frac{P_z(-\delta)}{P_z(+\delta)} = \frac{\delta^2 - |\epsilon|^2}{\delta^2 + |\epsilon|^2}$$

Thus the strength $|\epsilon|$ is also the "width" of the resonance.

For a linear crossing of the resonance we write $\delta \equiv \kappa - \kappa_o = a\theta$ and writing

$$\phi \equiv \begin{pmatrix} fe^{-ia\theta^2/4} \\ ge^{ia\theta^2/4} \end{pmatrix}$$

we obtain from Eq. (8).

$$\begin{cases} \dfrac{df}{d\theta} = \dfrac{i}{2}\,\epsilon g e^{\frac{i}{2}a\theta^2} \\[2em] \dfrac{dg}{d\theta} = \dfrac{i}{2}\,\epsilon^* f e^{-\frac{i}{2}a\theta^2} \end{cases} \tag{10}$$

The solutions of Eq. (10) are expressible in terms of confluent hypergeometric functions and give the Froissart-Stora[5] asymptotic relation for polarization

$$\frac{P_z(+\infty)}{P_z(-\infty)} = 2\exp(-\pi|\epsilon|^2/2a) - 1 \tag{11}$$

To cross the resonance without depolarization we can do either of two things:

1. Make $|\epsilon|^2/a \rightarrow 0$ and hence $P(+\infty) = P(-\infty)$. For intrinsic resonances $\gamma G = kS \pm \nu_z$ we can use pulsed quadrupoles to shift ν_z rapidly to obtain a fast crossing (large a). For imperfection resonances $\gamma G = k$ one uses trim dipoles to correct the vertical closed orbit distortion to reduce $|\epsilon|$. One can reduce $|\epsilon|$ also for intrinsic resonances using trim quadrupoles[6] but there are more conditions to be satisfied.

2. Make $|\epsilon|^2/a \rightarrow \infty$ and hence $P(+\infty) = -P(-\infty)$. This is the obvious complementary procedure of 1. The slow adiabatic passage of a strong resonance results in a total polarization flip.

Both procedures have been applied to obtain the polarized beams at ZGS[7], AGS[8], Saturne[9] and KEK-PS[10].

ELIMINATING RESONANCES BY SPIN ROTATORS (SIBERIAN SNAKES)

A. Principles and descriptions

The possibility of eliminating the resonances altogether for the 3-dimensional spin precessional motion was first pointed out by Derbenev and Kondratenko[11]. The principle of the method is simply illustrated for a 1-dimensional resonance in Fig. 2. The upper curve shows an oscillatory motion driven resonantly by the oscillatory force shown as the lower curve. If the phase of the motion could be shifted by 180° every k oscillations, the amplitude would then be alternately excited and damped and the resonance becomes ineffective.

In practice, the phase shifting is not possible for the 1-dimensional oscillation, but quite straightforward for the 3-dimensional spin motion. It is equivalent to simply precessing the spin by 180° about an axis in the horizontal plane. With the 180° spin rotation applied regularly (every revolution, say) the horizontal depolarizing precession will grow for one revolution being excited by the resonance and damp for the next revolution. The series of dipoles placed in a straight section of the ring lattice to produce the 180° spin rotation is known as the Siberian snake. We will refer to it here simply as the Spin Rotator, a more meaningful name. The spin rotator must, of course, leave the particle orbit exterior to the rotator unaffected. Inside the rotator which may be more than 10 m long there will be transverse excursions of the particle trajectory away from the original closed orbit. These excursions are larger at lower energies and make greater demands on the dipole apertures.

A pair of evenly spaced rotators with orthogonal rotating axes form a Double Rotator and is most useful. Such a pair could comprise of a longitudinal rotator with axis along $\hat{y}$ (known as Type 1 Siberian snake invented by K. Steffen and shown in Fig. 3) and a transverse rotator with axis along $\hat{x}$ (Type 2 Siberian snake invented by A. Turrin and shown in Fig. 4). Although a rotator with an arbitrary axis can be designed, this pair of longitudinal and transverse rotators is adequate for all practical purposes. Note that a solenoid acts as a longitudinal rotator. But the field of the solenoid must be proportional to γ and the non-zero focal action of the solenoid on the beam must be compensated.

All rotators shown here employ only horizontal and vertical dipoles. Clearly, dipoles with arbitrary roll angles can also be applied. In fact, helical rotators using dipoles with continuously advancing roll-angles have been proposed by Ya. S. Derbenev and by E. D. Courant. So far, no systematic procedure for the design of rotators has been found. Rotators are still invented by clever cutting and fitting.

The amount of horizontal precession induced by ϵ between rotators (the analog of the amplitude growth in the 1-dimensional illustration) should not get too large. This implies that for strong resonances more than one rotator is required. For weak resonances the illustration of Fig. 2 indicates that if the total horizontal precession in k passages of the rotator is tolerable the rotation per passage needs be only $> \pi/k$. Such a partially excited rotator first suggested by T. Roser, has the advantage of having smaller orbit excursions in the dipoles thereby reducing the aperture requirements at lower energies just when the resonances are weaker.

B.   Actions of rotators

We now examine the actions of the rotators quantitatively. The rotation matrix of a full transverse rotator is

$$M_o = e^{\frac{i}{2}\pi\sigma_x} = \cos\frac{\pi}{2} + i\sigma_x \sin\frac{\pi}{2} = i\sigma_x \tag{12}$$

With one such rotator the rotation matrix for one revolution is

$$M = M_{\text{half-turn}}\, M_o M_{\text{half-turn}}$$

$$= e^{\frac{i}{2}\pi(\vec{w}\cdot\vec{\sigma})}(i\sigma_x)e^{\frac{i}{2}\pi(\vec{w}\cdot\vec{\sigma})} \tag{13}$$

$$= i\sigma_x - 2\frac{\epsilon_R}{w}\sin\frac{\pi w}{2}(\cos\frac{\pi w}{2} + i\sigma_w\sin\frac{\pi w}{2})$$

where, as was already defined before

$$\vec{w} \equiv w\hat{w} = \epsilon_R\hat{x} - \epsilon_I\hat{y} - \delta\hat{z}$$

$$w \equiv |\vec{w}| = \sqrt{\delta^2 + |\epsilon|^2}, \qquad \sigma_w \equiv \hat{w}\cdot\vec{\sigma}$$

The equivalent one-turn precession angle θ is given by

$$\cos \frac{\theta}{2} = \frac{1}{2} \, \text{Tr}(M) = -\frac{\epsilon_R}{\omega} \sin \pi \omega = - \cos\lambda \, \sin\pi|\epsilon| \qquad (14)$$

where λ is the phase angle of $\epsilon \equiv |\epsilon|e^{i\lambda}$ and where the last expression is the maximum value at $\delta=0$. To reach resonance $\theta=0$ and the right-hand expression must reach unity. Even for $\lambda = \pi$ or $-\cos\lambda = 1$ no resonance can be reached if

$$|\epsilon| < \frac{1}{2} \qquad (15)$$

i.e. all resonances with $|\epsilon| < 1/2$ are eliminated. The same is true with a single longitudinal rotator with matrix $M_\pi = i\sigma_y$. (See article C. below for an explanation of the subscript on M.)

For a double rotator

$$M = M_\pi \, M_{\text{half-turn}} \, M_o \, M_{\text{half-turn}}$$

$$= (i\sigma_y) \, e^{\frac{i}{2}\pi(\vec{w}\cdot\vec{\sigma})} \, (i\sigma_x) \, e^{\frac{i}{2}\pi(\vec{w}\cdot\vec{\sigma})}$$

$$= i\sigma_z - 2\frac{\epsilon_R}{\omega}\sin\frac{\pi\omega}{2}(i\sigma_y\cos\frac{\pi\omega}{2} - \sigma_y\sigma_\omega\sin\frac{\pi\omega}{2}) \qquad (16)$$

and

$$\cos\frac{\theta}{2} = -2\frac{\epsilon_R}{\omega}\frac{\epsilon_I}{\omega}\sin^2\frac{\pi\omega}{2}$$

$$= -\sin 2\lambda \, \sin^2\frac{\pi|\epsilon|}{2} \qquad \text{(max. at } \delta=0) \qquad (17)$$

The condition for not reaching resonance even at $-\sin 2\lambda = 1$ is then

$$|\epsilon| < 1 \qquad (18)$$

As expected the double rotator is "twice as good" as a single rotator. Another advantage of the double rotator is that its "closed spin-trajectory" (analog of the closed orbit for the orbital motion) is mostly vertical. Hence the spins of a vertically polarized beam will stay near the closed spin-trajectory to form a "paraxial" dynamics.

C. Multiple rotators for strong resonances

For strong resonances with $|\epsilon| > 1$ we need more than one pair of double rotators. If we have N pairs the rotation matrix for each pair is

$$M_{pair} = (i\sigma_y) \ e^{\frac{i}{2} \frac{\pi}{N}(\vec{w} \cdot \vec{\sigma})} \ (i\sigma_x) \ e^{\frac{i}{2} \frac{\pi}{N}(\vec{w} \cdot \vec{\sigma})} \tag{19}$$

and we have

$$\cos \frac{\theta}{2N} = - \sin 2\lambda \ \sin^2 \frac{\pi|\epsilon|}{2N} \qquad (\text{max. at } \delta = 0) \tag{20}$$

The condition for not reaching resonance ($\theta=0$) at $-\sin 2\lambda = 1$ is modified to

$$|\epsilon| < N \tag{21}$$

from Eq. (18) for a single pair. Thus to eliminate all resonances with $|\epsilon|$ up to 10 in SSC one would need more than 10 pairs of double rotators evenly distributed around the circumference.

A look at the general arrangement[12] of multiple rotators of arbitrary types is illuminating. A rotator with an arbitrary axis in the horizontal plane may be considered as an α-rotation about $\hat{z}$ followed by a π-rotation about $\hat{x}$. The matrix of such a rotator is

$$M_\alpha = e^{\frac{i}{2}\pi\sigma_x} \ e^{\frac{i}{2}\alpha\sigma_z} = i(\sigma_x \cos \frac{\alpha}{2} + \sigma_y \sin \frac{\alpha}{2}) \tag{22}$$

namely a π-rotation about an axis oriented at angle $\alpha/2$ from the transverse axis $\hat{x}$. A general rotator is shown in Fig. 5.

In the most general arrangement of 2N rotators the no-coupling ($\epsilon=0$) vertical precession angle in one turn is

$$\theta = \Sigma \ (-1)^k \ (\alpha_k + \psi_k)$$

where ψ_k is the vertical precession angle due to the ring lattice preceding the kth rotator and is energy dependent. The alternating signs account for the π-rotation about $\hat{x}$ at each rotator. To make θ independent of energy we must have

$$\Sigma \ (-1)^k \ \psi_k = 0. \tag{23}$$

then we have

$$\theta = \Sigma \ (-1)^k \ \alpha_k \tag{24}$$

The arrangement of N evenly distributed pairs of transverse $(a=0)$ and longitudinal $(a=\pi)$ rotators corresponds to the case where all ψ_k are equal. Eq. (23) is then automatically satisfied and Eq. (24) gives $\theta = N\pi$.

D. Partial rotators for weak resonances

The one-revolution rotation matrix for a partial (precession angle = πa with a<1) transverse rotator is

$$M = e^{\frac{i}{2}\pi w \sigma_w} \; e^{\frac{i}{2}\pi a \sigma_x} \; e^{\frac{i}{2}\pi w \sigma_w}$$

and we have at $\delta = 0$

$$\cos\frac{\theta}{2} = \frac{1}{2}\,\mathrm{Tr}(M) = \cos\frac{\pi a}{2}\cos\pi|\epsilon| - \cos\lambda\,\sin\frac{\pi a}{2}\sin\pi|\epsilon| \; . \quad (25)$$

At $\lambda=\pi$ which maximizes this expression we have

$$\cos\frac{\theta}{2} = \cos\,\pi(\frac{a}{2}-|\epsilon|) \, . \quad (26)$$

the condition for not reaching resonance is then

$$|\epsilon| < \frac{a}{2} \quad (27)$$

agreeing with Eq. (15) at a = 1. This simply states that the rotator must be just strong enough to keep the "operating point" outside (twice) the width of the resonance. At low energies when the resonances are weak, the rotators can be turned on at low "strength" so that the orbit excursion which is expected to scale as a will not be excessively large. For example in the AGS the second lowest intrinsic resonance at $\gamma G = \nu_z = 8.75$ or $\gamma = 4.88$ is relatively strong with $|\epsilon| = 0.015$. A pair of partial double rotator with a = 0.03 will be more than adequate. The orbit excursion in a 1.8 tesla field transverse rotator is 2.6 m/$\beta\gamma$ = 55 cm unacceptable large, but at a = 0.03 the excursion is only 55 cm × 0.03 = 1.65 cm.

Let us examine the general scaling of the orbit excursion with energy. If we set a $\propto |\epsilon|$ then the orbit excursion in a partial rotator will scale as

$$\text{excursion} \propto |\epsilon|/\beta\gamma \propto \begin{cases} \beta^{-1} & \text{imperfection res.} \\ (\beta^3\gamma)^{-1/2} & \text{intrinsic res.} \end{cases} \quad (28)$$

At $\gamma = 1.5$ we have $\beta^{-1} = 1.34$ and $(\beta^3\gamma)^{-1/2} = 1.27$, both acceptable values. Thus, the rotator resonance elimination scheme is practicable for all energies above $\gamma = 1.5$. If more than one pair of rotators are needed at the upper end of the energy range, each rotator can be operated at even lower "strength" at the low energy end. Thus, one can reach down to even lower γ values without entailing excessively large orbit excursions.

Most of the analyses of the actions of rotators described throughout this Section have been double-checked by numerical tracking.

It is, by now, more than ten years since the spin rotator was first proposed, yet no experimental test of the scheme has been made. An experiment to test a solenoid longitudinal rotator is in progress[13, 14] on the cooling ring of the Indiana University Cyclotron Facility and will soon be conducted. We expect that it will provide the first experimental confirmation of the analyses.

The above discussions show that the spin rotator (Siberian snake) resonance elimination scheme is practical and effective from γ values below 1.5 up to arbitrarily high values limited only by the number of rotators used. For very strong and especially, overlapping resonances this is the only viable scheme to avoid depolarization. In designing future accelerators and storage rings it is advisable to have space provided in the magnet lattice to accommodate the rotators.

In addition to rotators, research and development efforts on polarized sources and polarimeters are also in great need. The intensity of existing polarized ion sources is some three orders of magnitude below that of unpolarized sources. High intensity and high degree of polarization sources, and fast and non-destructive polarization measurements are very desirable.

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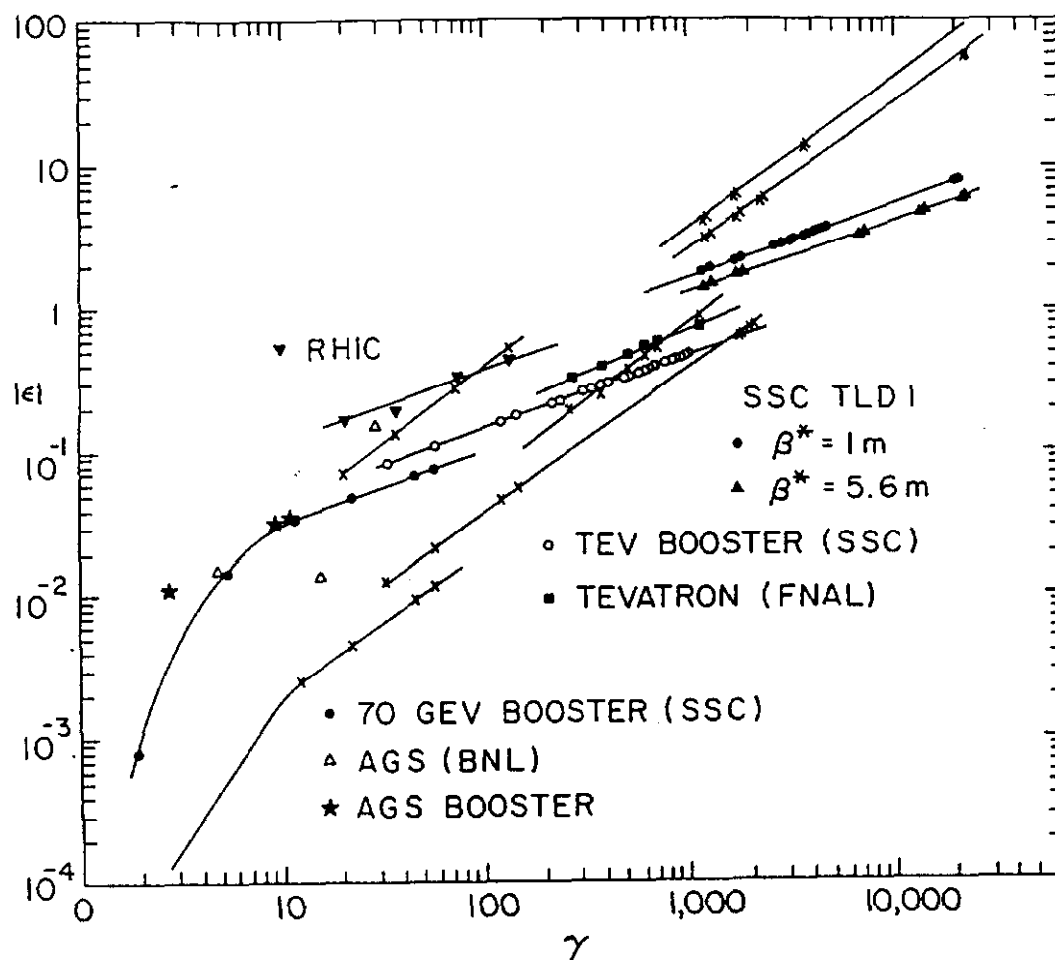
Figure 1. Strengths or widths $|\epsilon|$ of depolarizing resonances in several synchrotrons and storage rings plotted against energy γ. For intrinsic resonances (various symbols as denoted) the normalized invariant of the vertical oscillation is assumed to be 10π mm-mrad. For imperfection resonances (shown as crosses) the maximum alignment and gradient errors are assumed to be $\Delta x = \Delta z = 0.1$ mm and $\Delta B'/B' = 10^{-3}$.

OSCILLATION

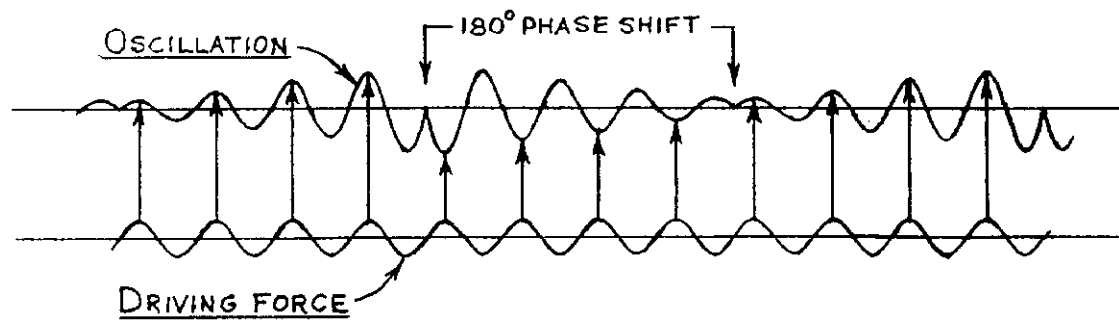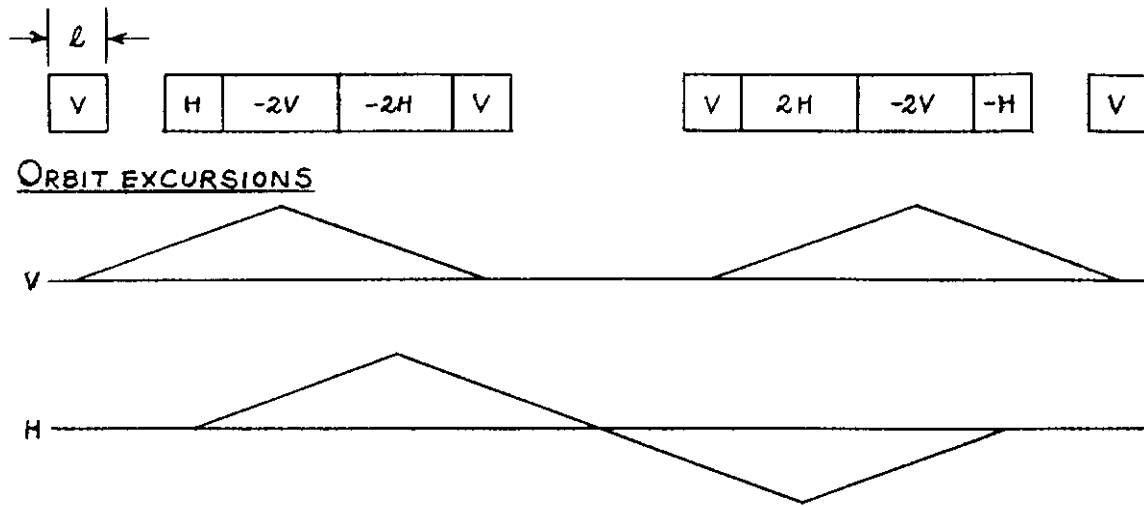180° PHASE SHIFT

DRIVING FORCE

Figure 2. A harmonic oscillation driven by a force in resonance. The motion shifts phase 180° every k (4 as shown) oscillations. The force alternately excites and damps the oscillation between phase shifts.

ORBIT EXCURSIONS

	Conventional dipole	Superconducting dipole
Field B	1.83 T	5.48 T
Unit length ℓ	0.75 m	0.25 m
Total length 19 ℓ	14.25 m	4.75 m
Orbit excursion $\Delta x = \Delta z$	1.15 m$/\beta\gamma$	0.38 m$/\beta\gamma$

Figure 3. Longitudinal rotator (Type 1 Siberian snake) precesses the spin 180° about the longitudinal ($\hat{y}$) axis. For the proton each unit has Bℓ = 1.37 Tm and precesses the spin 45°. H and V denote horizontal and vertical orbital deflections.
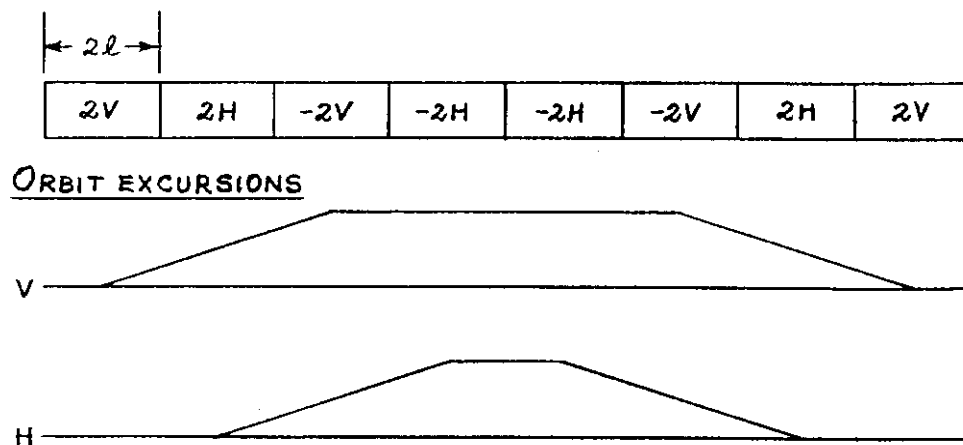
	⌐ 2ℓ ⌐							
	2V	2H	−2V	−2H	−2H	−2V	2H	2V

ORBIT EXCURSIONS

V

H

	Conventional dipole	Superconducting dipole
Field B	1.83 T	5.48 T
Unit length ℓ	0.75 m	0.25 m
Total length 16 ℓ	12 m	4 m
Orbit excursion $\Delta x = \Delta z$	2.63 m/$\beta\gamma$	0.88 m/$\beta\gamma$

Figure 4. Transverse rotator (Type 2 Siberian snake) precesses the spin 180° about the transverse ($\hat{x}$) axis. For the proton each unit has Bℓ = 1.37 Tm and precesses the spin 45°. H and V denote horizontal and vertical orbit deflections.

| H | | V | −2H | −V | | −V | 2H | V | | −H |

ORBIT EXCURSIONS

V

H
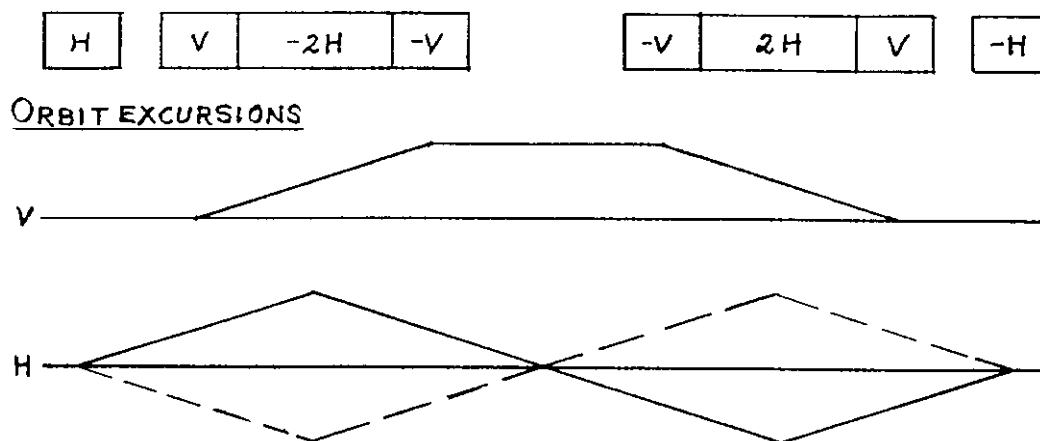
Precession axis angle $a/2$	$\pm75.0^{\circ}$	$\pm67.5^{\circ}$
V precession	54.7°	63.6°
H precession	60.0°	52.1°

Figure 5. Rotators with intermediate precession axes, neither transverse $(a/2=0)$ nor longitudinal $(a/2=90^{\circ})$. Rotators with other axial directions can also be designed.